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Towards a big crunch dual

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Abstract: We show there exist smooth asymptotically anti-de Sitter initial data which evolve to a big crunch singularity in a low energy supergravity limit of string theory. This opens up the possibility of using the dual conformal field theory to obtain a fully quantum description of the cosmological singularity. A preliminary study of this dual theory suggests that the big crunch is an endpoint of evolution even in the full string theory. We also show that any theory with scalar solitons must have negative energy solutions. The results presented here clarify our earlier work on cosmic censorship violation in $N = 8$ supergravity.

Keywords: AdS-CFT and dS-CFT Correspondence, Supergravity Models.

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Contents

1. Introduction

One of the main goals of quantum gravity is to provide a better understanding of the big bang or big crunch singularities in cosmology. Perhaps the most fundamental question is whether these singularities represent a true beginning or end of time (perhaps described by a special quantum state) [1], or whether there is some type of bounce, as envisioned by the pre-big bang [2] or cyclic universe models [3]. Since our usual notions of space and time are likely to break down near cosmological singularities, a particularly promising approach to study this issue might be to find a dual description in terms of more fundamental variables.

In string theory we do not yet have a dual description of real cosmologies, but we do have the celebrated AdS/CFT correspondence [4] which provides a non-perturbative definition of string theory on asymptotically anti-de Sitter (AdS) spacetimes in terms of a conformal field theory (CFT). The dual CFT description has been used to study the singularity inside black holes [5], which is analogous to a cosmological singularity. Although some progress in this direction has been made, the fact that the singularity is hidden behind an event horizon clearly complicates the problem. This is because the CFT evolution is dual to bulk evolution in Schwarzschild time so the CFT never directly 'sees' the singularity.

It would be better to have examples of solutions in a low energy supergravity limit of string theory where smooth, asymptotically AdS initial data evolve to a big crunch singularity. Then AdS/CFT should provide a precise framework in which the quantum nature of cosmological singularities could be understood, at least in asymptotically AdS spaces. In this context, a big crunch singularity is simply any spacelike singularity which extends to infinity and reaches the boundary in finite time.

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In this paper we present examples of such solutions in the familiar $\mathcal{N} = 8$, $D = 4$ supergravity obtained by compactifying eleven dimensional supergravity on S^7. This theory has scalars with $m^2 = -2$ in units of the AdS radius. It has recently been shown [6] that there is a one parameter family of boundary conditions for this field which preserve the full set of asymptotic AdS symmetries. When this parameter vanishes, the dual CFT is the usual $2+1$ theory on a stack of M2-branes. Nonzero values of the parameter correspond to modifying this theory by a triple trace operator. We will show that for all nonzero values of the parameter, there are bulk solutions where smooth, finite mass initial data evolve to a big crunch. We will also give a preliminary argument that evolution in the dual CFT also ends in finite time.

As a second class of examples, we briefly discuss the case of $\mathcal{N} = 8$, $D = 5$ supergravity. This theory has scalars with $m^2 = -4$ saturating the Breitenlohner-Freedman (BF) bound [7] in five dimensions. Once again, there is a one parameter family of AdS invariant boundary conditions [6, 8]. We find that for almost all values of this parameter, there are solutions which evolve to a big crunch. $\mathcal{N} = 8$ supergravity with these boundary conditions is similar to the theory discussed in [9] as a possible counterexample to cosmic censorship. Some valid criticism of that work was raised in [10] and [11], but the results presented here can be viewed as confirmation that with suitable boundary conditions, this theory admits solutions where initial data evolve to singularities that are not hidden inside event horizons.

Having asymptotically AdS initial data evolving to a big crunch, is clearly a sign of (nonlinear) instability. Other instabilities in the context of AdS/CFT have been discussed previously [12, 11].

We begin in the next section 2 with a general result about stability. We show that any theory of gravity coupled to a scalar field that has static solitons must also have solutions with negative total energy. In section 3 we discuss our main example, including a description of the boundary conditions and a demonstration that AdS_4 is unstable. We also discuss what is known about the dual CFT and make some preliminary remarks about the dual field theory description of the big crunch. Section 4 discusses the five dimensional example, and section 5 contains some concluding remarks. In the appendix we generalize some of our calculations to arbitrary dimensions and more general matter fields.

2. Scalar solitons imply negative energy

Consider gravity coupled to a scalar field with potential $V(\phi)$ in any spacetime dimension greater than three. In this section we prove the following result.

Claim. *If there exists a nonsingular, static, spherically symmetric, finite energy solution, then the positive energy theorem must be violated.*

In other words, if the theory has solitons, it must have negative energy solutions. Note that we are not claiming that the soliton itself must have negative energy, but only that negative energy solutions must exist. Partial versions of this result applying to non-negative potentials have already appeared in the literature. In particular, the argument below is a slight generalization of the scaling argument given in [13].

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We begin with the result that a static solution must extremize the energy. In the context of general relativity, this was shown in detail by Wald and Sudarsky [14]. Their argument is basically the following. The hamiltonian H is a function of the spatial metric g_{ij}, scalar field configuration ϕ, and their conjugate momenta π^{ij} and p. It takes the form of a volume integral of the constraints, plus a surface term. For any solution, the volume term vanishes and the surface term is the total mass. By definition, the variation of H gives the time derivatives of the fields. More precisely, for any perturbation $\delta g_{ij}, \delta\pi^{ij}, \delta\phi, \delta p$ one has

$$\delta H = \int -\dot{\pi}^{ij}\delta g_{ij} + \delta \dot{g}_{ij}\delta\pi^{ij} - \dot{p}\delta\phi + \dot{\phi}\delta p\,. \tag{2.1}$$

For a static solution, the time derivatives are all zero (assuming we choose the lapse and shift to correspond to the timelike Killing field). So the static solution is an extremum of H. If we choose the perturbation to satisfy the linearized constraints, then the volume contribution to δH vanishes. Therefore static solutions must extremize the surface term, i.e., the total mass.

The above argument was initially given for asymptotically flat spacetimes, but it also applies to asymptotically AdS spacetimes which we will explicitly consider below. So we assume $V(\phi)$ has a negative extremum which ϕ approaches asymptotically. To continue, we will need an explicit formula for the mass of spherically symmetric (and time symmetric) initial data when the scalar field has a profile $\phi(r)$. We will work in d spatial dimensions. In this case, the constraint equations reduce to (with $8\pi G = 1$)

$$^{d}\mathcal{R} = g^{ij}\phi_{,i}\phi_{,j} + 2V(\phi)\,. \tag{2.2}$$

The spatial metric can be written as

$$ds^2 = \left(1 - \frac{m(r)}{(d-1)r^{d-2}} + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega_{d-1}\,, \tag{2.3}$$

where ℓ is the radius of curvature of the asymptotic AdS space. It is related to the asymptotic value of the potential, Λ, by

$$\ell^2 = -\frac{d(d-1)}{2\Lambda}\,. \tag{2.4}$$

The constraint (2.2) then yields the following equation for $m(r)$

$$m_{,r} + \frac{1}{d-1}m(r)r(\phi_{,r})^2 = r^{d-1}\left[2(V(\phi)-\Lambda) + \left(1+\frac{r^2}{\ell^2}\right)(\phi_{,r})^2\right]. \tag{2.5}$$

The general solution for arbitrary $\phi(r)$ is

$$m(r) = \int_0^r e^{-\frac{1}{d-1}\int_{\tilde{r}}^{r} d\hat{r}\ \hat{r}(\phi_{,r})^2}\left[2(V(\phi)-\Lambda) + \left(1+\frac{\tilde{r}^2}{\ell^2}\right)\phi_{,\tilde{r}}^2\right]\tilde{r}^{d-1}d\tilde{r}\,. \tag{2.6}$$

The total mass is proportional to the asymptotic value of $m(r)$:

$$M = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}\lim_{r\to\infty} m(r)\,. \tag{2.7}$$

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Now suppose $\phi_0(r)$ is a static soliton and consider the one parameter family of configurations $\phi_\lambda(r) = \phi_0(\lambda r)$. Then from (2.6) and (2.7), it is easy to see that the total mass takes the form

$$M_\lambda = \lambda^{-d} M_1 + \lambda^{-(d-2)} M_2\,, \tag{2.8}$$

where M_2 is independent of the potential and is manifestly positive, and both M_i are independent of λ. Since the soliton extremizes the energy

$$0 = \frac{dM_\lambda}{d\lambda}|_{\lambda=1} = -dM_1 - (d-2)M_2\,. \tag{2.9}$$

Therefore, for all $d > 2$ one has $M_1 = -\frac{d-2}{d} M_2 < 0$. The potential contribution to the mass is negative. This means that the configuration $\phi_\lambda(r)$ with small λ has negative total mass. This argument clearly generalizes to several scalar fields and general non-linear sigma model type kinetic terms: $G_{ab}(\phi)\nabla_\mu \phi^a \nabla^\mu \phi^b$.

3. Main example

Our main example starts with the low energy limit of string theory with $AdS_4 \times S^7$ boundary conditions. The massless sector of the compactification of $D = 11$ supergravity on S^7 is $\mathcal{N} = 8$ gauged supergravity in four dimensions [15]. The bosonic part of this theory involves the graviton, 28 gauge bosons in the adjoint of $SO(8)$, and 70 real scalars, and admits AdS_4 as a vacuum solution. It is possible to consistently truncate this theory to include only gravity and a single scalar with action [16]

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + 2 + \cosh\left(\sqrt{2}\phi\right)\right]. \tag{3.1}$$

(We have chosen the gauge coupling so that the AdS radius is equal to one.) Notice that the potential is unbounded from below, and the scalar has mass

$$m^2 = -2\,. \tag{3.2}$$

Even though the field is tachyonic, the mass is above the Breitenlohner-Freedman bound $m^2_{BF} = -9/4$ [7], and with the usual boundary conditions AdS_4 is stable.

3.1 AdS invariant boundary conditions

We will mainly work in global coordinates in which the AdS_4 metric takes the form

$$ds_0^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2\,. \tag{3.3}$$

For $m^2 = -2$, solutions to the linearized wave equation $\nabla^2\phi - m^2\phi = 0$ with harmonic time dependence $e^{-i\omega t}$ all fall off asymptotically like

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2}\,. \tag{3.4}$$

The standard boundary conditions correspond to taking $\alpha = 0$ in (3.4), so the modes fall off as fast as possible. For those boundary conditions, the total energy is finite, using the standard definition of energy [17]. Recall that the energy, and more generally, conserved charges associated with asymptotic symmetries ξ^μ can be defined as follows [18]. One starts with the hamiltonian $H[\xi] = \int d^3x \xi^\mu H_\mu$ where H_μ are the usual constraints, adds surface terms so that H has well defined functional derivatives, and then subtracts the analogous expression for the AdS_4 background. This gives

$$Q_G[\xi] = \frac{1}{2} \oint dS_i \bar{G}^{ijkl}(\xi^\perp \bar{D}_j h_{kl} - h_{kl}\bar{D}_j \xi^\perp) + 2 \oint dS_i \frac{\xi^j \pi^i{}_j}{\sqrt{\bar{g}}}\,, \tag{3.5}$$

where $G^{ijkl} = \frac{1}{2} g^{1/2}(g^{ik}g^{jl} + g^{il}g^{jk} - 2g^{ij}g^{kl})$, $h_{ij} = g_{ij} - \bar{g}_{ij}$ is the deviation from the spatial metric $\bar{g}_{ij}$ of pure AdS, $\bar{D}_i$ denotes covariant differentiation with respect to $\bar{g}_{ij}$ and $\xi^\perp = \xi^\mu n_\mu$ with n_μ the unit normal to the surface.

However since $m^2 = -2$ lies in the range $m^2_{BF} + 1 > m^2 > m^2_{BF}$ there exists an additional one-parameter family of AdS invariant boundary conditions on the scalar field and the metric components [6]. More precisely, the asymptotic AdS symmetries are preserved in solutions that belong to the following class,

$$\phi(r,t,x^a) = \frac{\alpha(t,x^a)}{r} + \frac{f\alpha^2(t,x^a)}{r^2} \tag{3.6}$$

$$\begin{aligned}
g_{rr} &= \frac{1}{r^2} - \frac{(1+\alpha^2/2)}{r^4} + O\left(\frac{1}{r^5}\right) \qquad g_{tt} = -r^2 - 1 + O\left(\frac{1}{r}\right)\\
g_{tr} &= O\left(\frac{1}{r^2}\right) \qquad g_{ab} = \bar{g}_{ab} + O\left(\frac{1}{r}\right)\\
g_{ra} &= O\left(\frac{1}{r^2}\right) \qquad g_{ta} = O\left(\frac{1}{r}\right),
\end{aligned} \tag{3.7}$$

where $x^a = \theta, \phi$ and f is an arbitrary constant that labels the different boundary conditions. Notice that the boundary conditions on some of the metric components are also relaxed compared to the standard set. When $f = 0$ we recover the $\beta = 0$ boundary conditions which have been considered before in the context of AdS/CFT [19]. Remarkably, however, the asymptotic anti-de Sitter symmetries are preserved for all values of f. In particular, it is easy to see that rescaling r leaves f unchanged. Since α depends on the particular solution and can vanish, each of these boundary conditions admits AdS_4 as a solution.

For these more general boundary conditions, the usual energy (3.5) diverges. However, one can define finite conserved charges by repeating the above procedure. The net result is that there is an additional surface term involving the scalar field[1] [6]

$$Q[\xi] = Q_G[\xi] + \frac{1}{6}\oint \xi^\perp \left[(\nabla\phi)^2 - m^2\phi^2\right]. \tag{3.8}$$

[1]The form of the surface term given here is different from that in [6], but it is equivalent. Both the divergent and finite terms agree. On the other hand, the finite contribution differs from the energy defined by holographic renormalization, as discussed in [20, 21], since they require that the energy vanishes for a nontrivial domain wall background.

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For all finite f (including $f = 0$!) the scalar and gravitational charge separately diverge. The divergences, however, exactly cancel out yielding finite total charges $Q[\xi]$. By contrast, the scalar charges Q_ϕ vanish for the standard boundary conditions (3.4) with $\alpha = 0$.

For spherical solutions it is easy to compute the total mass $M = Q[\partial_t]$, which yields

$$M = 4\pi \left(M_0 + \frac{4}{3} f \alpha^3 \right), \tag{3.9}$$

where M_0 is the coefficient of the $O(1/r^5)$ correction to g_{rr}.

3.2 Solitons

We now look for static, spherically symmetric asymptotically AdS soliton solutions. Writing the metric as

$$ds^2 = -h(r) e^{-2\delta(r)} dt^2 + h^{-1}(r) dr^2 + r^2 d\Omega_2 \tag{3.10}$$

the field equations read

$$h\phi_{,rr} + \left(\frac{2h}{r} + \frac{r}{2}\phi_{,r}^2 h + h_{,r} \right) \phi_{,r} = V_{,\phi} \tag{3.11}$$

$$1 - h - r h_{,r} - \frac{r^2}{2}\phi_{,r}^2 h = r^2 V(\phi) \tag{3.12}$$

$$\delta_{,r} = -\frac{r\phi_{,r}^2}{2}. \tag{3.13}$$

Regularity at the origin requires $h = 1$, $h_{,r} = 0$ and $\phi_{,r} = 0$ at $r = 0$.

We have numerically solved these equations. For every nonzero ϕ_0 at the origin, the solution to (3.11) is asymptotically of the form (3.6) for some value of f. The staticity and spherical symmetry of the soliton mean $\alpha(t, x^a)$ is simply a constant. The scalar field value ϕ_0 at the origin uniquely determines f and vice versa; there is at most one static spherical soliton solution in each theory. We find a regular soliton solution for all finite $f \neq 0$. When $|f| \to 0$ we find $|\phi_0| \to \infty$ and for $|f| \to \infty$ one has $|\phi_0| \to 0$ so the nontrivial soliton solution ceases to exist in this limit. As an example, in figure 1 we show the soliton solution for $f = -.25$ boundary conditions, which has $\phi_0 \approx 1.5$.

The existence of soliton solutions for a large class of AdS invariant boundary conditions implies there are negative mass solutions in those theories. This is because the soliton solutions extremize the energy (3.9). Although the total mass M now acquires a contribution from the scalar field, the argument given in section 2 still applies: the total mass consists of a finite term M_1 that scales as the volume (which includes the scalar contribution) and a finite, positive term M_2 that scales linearly in r. Hence M_1 must be negative for the soliton, which means rescaled configurations $\phi_\lambda(r) = \phi_0(\lambda r)$ with sufficiently small λ must have negative mass.

Such rescaled configurations are initial data for time-dependent solutions. By adjusting λ, one can arrange to have an arbitrarily large central region where ϕ is essentially constant and away from the maximum of the potential. It follows that the field must evolve to a spacelike singularity [9]. Moreover, the singularity that develops cannot be hidden behind

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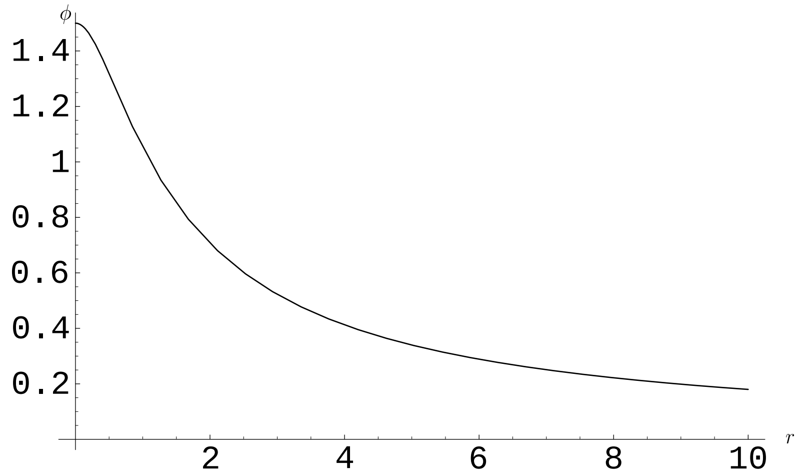

Figure 1: Soliton solution $\phi(r)$ with boundary conditions specified by $f = -1/4$.

an event horizon, because all spherically symmetric black holes have positive mass [6]. Instead, one expects it to continue to spread, cutting off all space.[2] The existence of solitons, therefore, indicates the existence of finite mass configurations that produce a big crunch. A particular example of such a configuration for which this can be shown explicitly is given next.

3.3 Instantons

First we construct an $O(4)$-invariant euclidean instanton solution of the form

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3 \tag{3.14}$$

and $\phi = \phi(\rho)$. The field equations determine b in terms of ϕ

$$b^2(\rho) = \frac{2V\rho^2 - 6}{\rho^2\phi'^2 - 6} \tag{3.15}$$

and the scalar field ϕ itself obeys

$$b^2\phi'' + \left(\frac{3b^2}{\rho} + bb'\right)\phi' - V_{,\phi} = 0\,, \tag{3.16}$$

where prime denotes ∂_ρ. Regularity again requires $\phi'(0) = 0$.

From (3.16) it follows that asymptotically $\phi(\rho)$ has the same behavior as the lorentzian scalar field solutions considered above,

$$\phi = \frac{\alpha}{\rho} + \frac{f\alpha^2}{\rho^2}\,. \tag{3.17}$$

[2] If V were bounded from below, it has been shown that the singularity cannot end or become timelike [22]. The same is likely to be true here.

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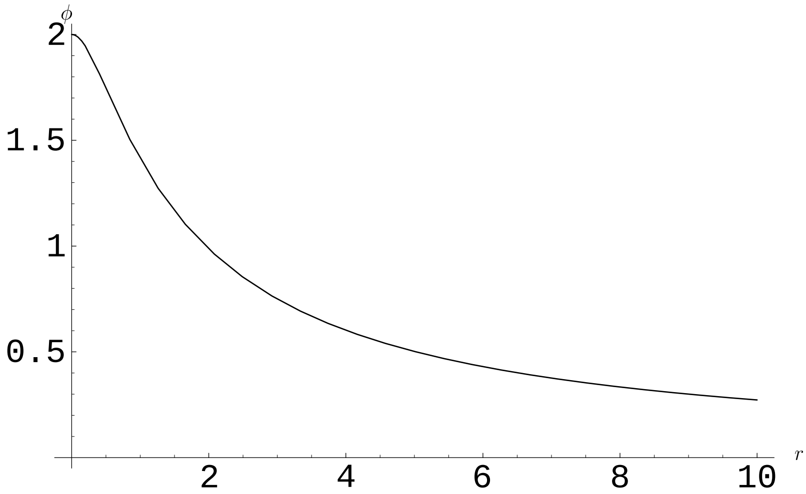

Figure 2: Instanton solution $\phi(\rho)$ with boundary condition $f = -1/4$.

We find that all boundary conditions that admit a spherical soliton solution also admit an $O(4)$-invariant instanton solution. As for the solitons, f is determined by the field $\phi(0)$ at the origin. In figure 2, the profile $\phi(\rho)$ is shown of the instanton with $f = -.25$ boundary conditions.

The solution we want is obtained by analytically continuing the instanton. We will discuss this in the next subsection, but first we comment on the interpretation of this instanton. The existence of negative energy solutions indicates that AdS_4 is unstable with these boundary conditions. The quantum decay rate is determined in a semiclassical approximation by the euclidean action of instanton. The action is given by

$$I = \int \left[-\frac{1}{2}R + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right] - \oint K + \frac{1}{6} \oint \left[(\nabla\phi)^2 - m^2\phi^2 \right], \tag{3.18}$$

where the first surface term is the usual Gibbons[1]-Hawking term, and the second is the surface term required so that the hamiltonian constructed from this action (after subtracting the background) agrees with (3.8). For the case $f = 0$ this term is equivalent to the surface term $-\frac{1}{2}\oint \phi \nabla_i \phi dS^i$ introduced by Klebanov and Witten to render the action of the α/r modes finite [19].

The relevant quantity for computing the rate of vacuum decay is the difference between the instanton action and the action for pure AdS: $\Delta I = I - I_{\text{AdS}}$. Subtracting I_{AdS} removes the leading divergences in I, but since ϕ goes to zero so slowly, there are two subleading divergences. If the coefficients of these terms were not exactly zero, ΔI would be infinite and there would be no probability for the vacuum to decay. In the appendix we show that both coefficients miraculously vanish. This involves nontrivial cancellations among the volume term and both surface terms in the action. Furthermore, the difference ΔI becomes small for large $|f|$ and goes to zero when $|f| \to \infty$.

3.4 Big crunch instability

Consider the slice through the instanton obtained by restricting to the equator of the S^3. The fields on this surface define time symmetric initial data for a lorentzian solution. The euclidean radial distance ρ simply becomes the radial distance r in the lorentzian solution. The total mass (3.9) of this initial data can be computed from the instanton geometry. Substituting (3.17) into (3.15) yields asymptotically

$$b^2(\rho) = \rho^2 + 1 + \frac{\alpha^2}{2} + \frac{4f\alpha^3}{3\rho}\,. \tag{3.19}$$

This is of the form (3.7) required to have finite conserved charges. In fact, we see that $M_0 = -4f\alpha^3/3$ and hence (3.9) implies that the total mass is zero! This is consistent with its interpretation as the solution AdS_4 decays into.

The evolution of this initial data is simply obtained by analytic continuation of the instanton. This is discussed in detail in [23], but the basic idea is the following. The origin of the euclidean instanton becomes the lightcone of the lorentzian solution. Outside the lightcone, the solution is given by (3.14) with $d\Omega_3$ replaced by three dimensional de Sitter space. The scalar field ϕ remains bounded in this region. Inside the lightcone, the $SO(3,1)$ symmetry ensures that the solution evolves like an open FRW universe,

$$ds^2 = -dt^2 + a^2(t)d\sigma_3\,, \tag{3.20}$$

where $d\sigma_3$ is the metric on the three dimensional unit hyperboloid. The field equations are

$$\frac{\ddot{a}}{a} = \frac{1}{3}[V(\phi) - \dot{\phi}^2] \tag{3.21}$$

$$\ddot{\phi} + \frac{3\dot{a}}{a}\dot{\phi} + V_{,\phi} = 0 \tag{3.22}$$

and the constraint equation is

$$\dot{a}^2 - \frac{a^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] = 1\,, \tag{3.23}$$

where $\dot{a} = \partial_t a$. On the light cone, $\phi = \phi(0)$ and $\dot{\phi} = 0$ (since $\phi_{,\rho} = 0$ at the origin in the instanton). Under evolution ϕ rolls down the negative potential, so the right hand side of (3.21) decreases. This ensures that $a(t)$ vanishes in finite time producing a big crunch singularity. For the purpose of understanding cosmological singularities in string theory, one can forget the origin of this solution as the analytic continuation of an instanton. We have simply found an explicit example of asymptotically AdS initial data which evolves to a big crunch.

The discussion above is closely analogous to the situation in theories with a true vacuum as well as a false vacuum, in which one can describe the decay of the false vacuum by nucleating a bubble of true vacuum. One typically finds that small bubbles of true vacuum collapse, large bubbles expand, and a critical size bubble is static. Our soliton is like a static critical bubble. The solution obtained from the instanton is like a larger bubble which expands. This can be seen by comparing figure 1 and figure 2. The field profile obtained from the instanton is indeed broader than the static soliton.

3.5 Dual CFT description

Having shown that the bulk theory admits solutions which evolve to a big crunch, we now turn to the dual CFT description of this theory. The dual to string theory on $AdS_4 \times S^7$ can be obtained by starting with the field theory on a stack of N D2-branes. This is a SU(N) gauge theory with seven adjoint scalars φ^i. One then takes the infrared (strongly coupled) limit to obtain the CFT. In the process, one obtains an $SO(8)$ symmetry. In the abelian case, $N = 1$, this can be understood by dualizing the three dimensional gauge field to obtain another scalar. But in general, it is not well understood.

This theory has dimension one operators $\mathcal{O}_T = TrT_{ij}\varphi^i\varphi^j$ where T_{ij} is symmetric and traceless [24].[3] One of these, $\mathcal{O}$, is dual to the bulk field we have been considering with the boundary conditions that $\phi = \alpha/r + O(r^{-3})$ for physical states. The field theory dual to the "standard" quantization, where physical states are described by modes with $\phi = \beta/r^2$ asymptotically, can be obtained by adding the double trace term $\frac{f}{2}\int \mathcal{O}^2$ to the action [25, 26]. This is a relevant perturbation and the infrared limit is another CFT in which $\mathcal{O}$ has dimension two.

As described in [6], the AdS invariant boundary conditions correspond instead to adding a triple trace term to the action

$$S = S_0 + \frac{f}{3}\int \mathcal{O}^3\,. \tag{3.24}$$

This follows from Witten's treatment of multi-trace operators in AdS/CFT [25]. The extra term has dimension three, and hence is marginal and preserves conformal invariance, at least to leading order. One might wonder if this symmetry is exact, or whether the operator $\mathcal{O}^3$ has an anomalous dimension. The anomalous dimension can receive contributions proportional to $1/N$ or f. Since the large N limit corresponds to supergravity in the bulk with AdS invariant boundary conditions, and for every f there is a bulk solution corresponding to pure AdS, it seems likely that the theory remains conformally invariant for finite f (at least for large N).[4]

Since the lorentzian solution obtained from the instanton takes the form (3.14) with $d\Omega_3$ replaced by three dimensional de Sitter space, dS_3, one might think that the natural dual would correspond to the CFT on dS_3. This field theory certainly allows evolution for infinite time and is nonsingular. But this only corresponds to evolution for finite global time. We want to conformally rescale dS_3 to (part of) the cylinder $R \times S^2$. This is equivalent to a coordinate transformation in the bulk. The relation between the usual static coordinates (3.3) for AdS_4 and the $SO(3,1)$ invariant coordinates

$$ds^2 = \frac{d\rho^2}{1+\rho^2} + \rho^2(-d\tau^2 + \cosh^2\tau d\Omega_2) \tag{3.25}$$

is

$$\rho^2 = r^2\cos^2 t - \sin^2 t\,. \tag{3.26}$$

[3] Since there are only seven φ's and the theory has $SO(8)$ symmetry, there are other operators involving the gauge field which complete the $SO(8)$ representation.

[4] This is different from the example considered in [27] which also had a bulk AdS solution, but imposed boundary conditions that were not AdS invariant.

Since our bulk solution asymptotically has

$$\phi(\rho) = \frac{\alpha}{\rho} + \frac{f\alpha^2}{\rho^2} + O(\rho^{-3})\,. \tag{3.27}$$

This becomes

$$\phi(r) = \frac{\tilde{\alpha}}{r} + \frac{f\tilde{\alpha}^2}{r^2} + O(r^{-3})\,, \tag{3.28}$$

where $\tilde{\alpha} = \alpha/\cos t$. Notice that f is unchanged. Hence the evolution of the initial data that we described in section 3.4 preserves the AdS invariant boundary conditions (3.6)–(3.7). This also provides some support for the fact that (3.24) is conformally invariant for finite f. The fact that $\tilde{\alpha}$ blows up as $t \to \pi/2$ is consistent with the fact that this is the time that the big crunch singularity hits the boundary. Since the coefficient of $1/r$ is usually interpreted as the expectation value of $\mathcal{O}$ in the CFT, this also indicates that the CFT on the cylinder does NOT have well defined evolution for all time.

A qualitative explanation for this is the following. The term we have added to the action is not positive definite. Since the energy associated with the asymptotic time translation in the bulk can be negative, the dual field should also admit negative energy states. This strongly suggests that the usual vacuum is unstable. It might decay via the (nongravitational) decay of the false vacuum. Perhaps a useful analogy is a scalar field theory with potential $V = m^2\varphi^2 - f\varphi^6$. The quadratic term is analogous to the coupling of φ to the curvature of S^2, which is needed for conformal invariance. The second term is analogous to the second term in (3.24). Qualitatively this theory has the same behavior as the bulk. There are instantons which describe the semiclassical decay of the usual vacuum $\varphi = 0$. For small f, the potential barrier is large, and the instanton action is large. So tunneling is suppressed. For large f, the barrier is small and tunneling is not suppressed. After the tunneling, the field rolls down the potential and becomes infinite in finite time. So in the semiclassical description of this analogous field theory, evolution ends in finite time. If the same is true in the full description of the theory (3.24) one could conclude that there is no bounce through the big crunch singularity in the bulk.

We close this section with a few comments about generalizations. We have focused in this section on big crunch singularities, but the solutions we have presented have a big bang singularity in the past as well, which should have a dual CFT description too. In addition, it is likely there exist solutions with only one singularity, in the future or the past. We have also restricted ourselves to gravity coupled to a single scalar, but this was just for simplicity. We expect that with suitable boundary conditions, there are more general bulk solutions to low energy string theory that form a big crunch which include several scalars, and possibly other matter fields.

4. A big crunch in $D = 5$ supergravity

$\mathcal{N} = 8$ gauged supergravity in five dimensions [28, 29] is thought to be a consistent truncation of ten dimensional type IIB supergravity on S^5. The spectrum of this compactification involves 42 scalars parameterizing the coset $E_{6(6)}/\mathrm{USp}(8)$. We will focus on a restriction of this theory to only gravity and one scalar [30]. The action is

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + \left(2e^{2\phi/\sqrt{3}} + 4e^{-\phi/\sqrt{3}}\right)\right]. \tag{4.1}$$

(We have again adjusted the gauge coupling so that the AdS radius is one.) As before, the potential is unbounded from below. Since the analysis in this section is similar to that in the previous section, our discussion will be brief.

The scalar ϕ has mass

$$m^2 = -4\,. \tag{4.2}$$

Since this saturates the BF bound in five dimensions, ϕ asymptotically behaves as

$$\phi(t,r,x^a) = \frac{\alpha(t,x^a)\ln r}{r^2} + \frac{\beta(t,x^a)}{r^2}\,. \tag{4.3}$$

In [6, 8] it was shown that the asymptotic AdS symmetry group is preserved for solutions with the following asymptotic behavior,

$$\beta = \alpha\left(f - \frac{1}{2}\ln\alpha\right) \tag{4.4}$$

together with appropriate boundary conditions on the metric components. Here f is again an arbitrary constant. For solutions within this class there exists a well-defined notion of mass, namely the conserved charge $M = Q[\xi]$ associated with the asymptotic Killing vector $\xi = \partial_t$. As before, this is finite but acquires a contribution from the scalar field. For spherical solutions it is given by (see (A.17) in the appendix)

$$M = 2\pi^2\left[\frac{3}{2}M_0 + \frac{1}{4}\alpha^2(\ln\alpha)^2 + \alpha^2\left(\frac{1}{4} - f\right)\ln\alpha + \alpha^2\left(f^2 - \frac{1}{2}f + \frac{1}{8}\right)\right], \tag{4.5}$$

where M_0 is now the coefficient of the $O(1/r^6)$ correction to the g_{rr} component of the metric.

To see that there are solutions which evolve to a big crunch we again start with the instanton. For finite f boundary conditions (which allow a logarithmic scalar mode) there exist regular $O(5)$-invariant euclidean instanton solutions,

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_4 \tag{4.6}$$

with $\phi = \phi(\rho)$ a solution of

$$b^2\phi'' + \left(\frac{4b^2}{\rho} + bb'\right)\phi' - V_{,\phi} = 0\,. \tag{4.7}$$

An example of an instanton solution with $f = 1$ boundary conditions is shown in figure 3. All finite f boundary conditions admit precisely one instanton solution. The analytic continuation of the instanton again describes asymptotically AdS initial data evolving to a big crunch. In the appendix we show that the total mass (4.5) of this initial data vanishes. On the other hand, the mass of static, spherical black holes that are solutions with the same

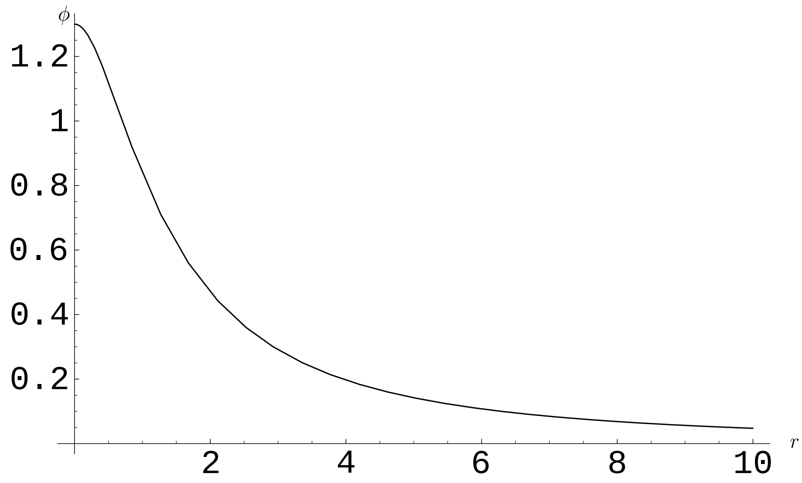

Figure 3: Instanton solution $\phi(\rho)$ with boundary condition $f = 1$.

boundary conditions is positive [6]. This shows that the singularity that develops from the initial data cannot be hidden behind an event horizon. Indeed, the explicit solution shows that, with generalized AdS invariant boundary conditions, the singularity reaches infinity in finite time.

If one requires $\alpha = 0$ in (4.3) the logarithmic mode is switched off. For this choice of boundary conditions we do not find an instanton. There is, however, still a finite scalar contribution to the conserved charges.[5] This means the standard gravitational mass that appears in the metric need not be positive, even though the scalar modes fall off as fast as possible. Examples of initial data with negative gravitational mass were given in [9]. At first sight this suggests $\alpha = 0$ boundary conditions may also admit solutions in which smooth, finite mass initial data evolve to a big crunch. However, the gravitational mass alone is generally not conserved during evolution [9] (this has also been observed in the numerical work of [10]). Instead, the quantity that is conserved during evolution is the total charge $Q[\partial_t]$. This is always positive [31] and we have not found any example where a big crunch is produced from smooth asymptotically AdS initial data with these boundary conditions.

We have previously argued [9] that cosmic censorship is generically violated in $\mathcal{N} = 8$, $D = 5$ supergravity when one allows the logarithmic mode for one or more scalars saturating the BF bound. We claimed that one could produce singularities that were not contained inside a black hole. Our argument consisted of trying to find initial data that evolve to a singularity in a central region, but do not have enough mass to form a black hole large enough to enclose the singular region.[6] Instead, we argued, the singularity must have a

[5] This is also evident in the spinorial proof [31] of the positive energy theorem for these boundary conditions, where the positive surface term (which equals $Q[\xi]$) contains an extra finite scalar contribution [9].

[6] In [9] we imposed a large radius cutoff at R_1 and required $\phi(R_1)$ to be constant in time to regularize the gravitational mass. This enabled us to compare the mass of the initial data with the mass required to form a (hairy) black hole obeying the same boundary condition at R_1. Working with a finite radius cutoff is of course somewhat artificial because this breaks asymptotic AdS invariance. The cutoff is not needed,

naked endpoint or extend all the way to infinity, like a big crunch. In both cases one might argue that cosmic censorship is violated since one cannot evolve for all time in the asymptotic region. Our conclusions have been questioned in [10] and [11], and indeed the examples we gave in [9] do have enough mass to form a black hole. Nevertheless, the results of this section show that our basic conclusion was essentially correct. The analytic continuation of the instanton shows that, for $\mathcal{N}=8$, $D=5$ supergravity with general AdS invariant boundary conditions, there are indeed smooth initial data which evolve to big crunch singularities. This is perhaps less surprising in light of the fact that the positive energy theorem is violated with these boundary conditions.

From the current standpoint of understanding big crunch singularities, this example is less satisfactory than the one discussed in the previous section. This is because it is not clear what the CFT deformations are that correspond to general AdS invariant boundary conditions in this case. There is a dimension two operator which is dual to our scalar field with boundary conditions (4.3) with $\alpha=0$. A naive application of Witten's prescription [25] involves taking the logarithm of this operator.

5. Conclusion

Motivated by a desire to find a dual quantum description of a cosmological singularity, we have constructed low energy string theory solutions that describe the evolution of asymptotically AdS initial data to a big crunch. Our main example involves a negative m^2 scalar field in $\mathcal{N}=8$, $D=4$ supergravity. To induce the instability, we have modified the boundary conditions on the scalar at infinity[7]. Although the modified boundary conditions preserve the full set of asymptotic AdS symmetries and allow a finite conserved energy to be defined, this energy can be negative. We have shown this by explicitly constructing negative energy initial data (the rescaled solitons of section 3.2). We have also constructed finite action instantons in this theory which describe the semiclassical decay of AdS_4. Most importantly, the analytic continuation of the instanton describes smooth (zero energy) initial data evolving to a big crunch.

We have also given an example of a similar solution in $\mathcal{N}=8$ supergravity in five dimensions, involving a scalar field saturating the BF bound. More generally, we show in the appendix that a big crunch can be produced from smooth initial data whenever one has a scalar field with $m^2_{BF} \leq m^2 < m^2_{BF}+1$ which decouples from the rest of the matter.

Usually one discards unstable theories, saying they are not of physical interest. Here, we are using the fact that there should be a dual CFT description of these bulk theories even if they are unstable. The field theory should provide a complete quantum description of the big crunch singularity. Clearly the next step is to try to better understand these dual

however, for the class of solutions considered here, because the boundary conditions are AdS invariant despite the logarithmic branch. The comparison of the mass, therefore, can be done using the conserved charge (4.5). We note, however, that the evolution with fixed f is likely to differ from the evolution with fixed $\phi(R_1)$.

[7]This is a nonlinear instability since, at the linearized level, the boundary conditions (3.6) reduce to the $\beta=0$ case discussed in [7].

theories. If states in the CFT have a well defined evolution for all time, and a semiclassical bulk metric can be reconstructed at late time, then there must be a bounce through the singularity. However, if the CFT evolution ends after finite time, or a semiclassical metric cannot be constructed, then the bulk evolution would end at the big crunch.

In $\mathcal{N} = 8$, $D = 4$ supergravity we find solutions which evolve to a big crunch for a one parameter family of AdS invariant boundary conditions. These solutions do not exist when the parameter vanishes, in which case the dual CFT is the usual $2+1$ theory on a stack of M2-branes. Nonzero values of the parameter correspond to modifying this field theory by a triple trace operator. Since the energy associated with asymptotic time translations in the bulk can be negative, AdS/CFT predicts this dual field theory should also admit negative energy states. This is plausible, since the extra term we have added to the CFT action is not positive definite. We have given preliminary evidence that for some states, the CFT does not have well defined evolution for all time (at least in the large N limit). The dual CFT corresponding to $\mathcal{N} = 8$, $D = 5$ supergravity with generalized boundary conditions is understood even less well, but investigations in a related context [11] indicate it exhibits similar instabilities.

Furthermore, we have evidence that bulk solutions cannot evolve to a big crunch in supergravity theories that are dual to stable CFT's. In [32, 33] we considered gravity coupled to scalar fields with potential satisfying the positive energy theorem (in AdS).[8] We looked for finite mass initial data which evolve to a singularity but do not have enough mass to enclose the singular region inside a black hole. We did not find any such examples. Taken together, these results suggest the big crunch solutions given here *require* boundary conditions that correspond to an unstable dual CFT. It is then plausible that there will be certain CFT states which do not have well defined evolution for all time. We have seen this happening at the semiclassical level in the deformed $2+1$ theory in dual description of our big crunch solutions. This is suggestive of the big crunch being an endpoint of evolution even in the full string theory.[9]

This would raise the question what determines the boundary conditions at cosmological singularities. Perhaps the AdS/CFT correspondence and the toy models of cosmologies we have constructed here could be useful to study this question further.

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[8] These potentials can always be derived from a superpotential $W(\phi)$, where $V = (d-1)W'^2 - dW^2$ and ϕ approaches an extremum of W at infinity [34].

[9] It is possible that the big crunch singularities that we have been studying in AdS will turn out to have qualitative differences from true cosmological singularities.

A. Generalization to arbitrary dimension

We consider a scalar field with potential $V(\phi)$ coupled to gravity in $d+1$ dimensions. We assume that $\phi = 0$ is a negative local maximum for the potential (with value corresponding to a unit AdS radius) and the field has mass $m_{BF}^2 < m^2 < m_{BF}^2 + 1$. Suppose there is a euclidean instanton solution of the form

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_d \tag{A.1}$$

and $\phi = \phi(\rho)$. Then the field equations determine b in terms of ϕ

$$b^2(\rho) = \frac{2V\rho^2 - d(d-1)}{\rho^2\phi'^2 - d(d-1)}\,. \tag{A.2}$$

The scalar field ϕ itself obeys

$$b^2\phi'' + \left(\frac{db^2}{\rho} + bb'\right)\phi' - V_{,\phi} = 0 \tag{A.3}$$

with $\phi'(0) = 0$. Therefore, asymptotically ϕ has the same behavior as the lorentzian scalar field solutions of the same mass

$$\phi = \frac{\alpha}{\rho^{\Delta_-}} + \frac{\beta}{\rho^{\Delta_+}}\,, \tag{A.4}$$

where

$$\Delta_\pm = \frac{d \pm \sqrt{d^2 + 4m^2}}{2}\,. \tag{A.5}$$

For instanton solutions, the coefficients α and β are determined by $\phi(0)$. Substituting (A.4) into (A.2) yields asymptotically

$$b^2(\rho) = \rho^2 + 1 + \frac{\Delta_-\alpha^2}{(d-1)}\frac{1}{\rho^{2\Delta_- - 2}} + \frac{4\Delta_-\Delta_+\alpha\beta}{d(d-1)}\frac{1}{\rho^{d-2}}\,. \tag{A.6}$$

We now compute the euclidean action of the instanton,

$$I = \int [-\frac{1}{2}R + \frac{1}{2}(\nabla\phi)^2 + V(\phi)] - \oint K + \frac{1}{2d}\oint\left[(\nabla\phi)^2 - m^2\phi^2\right], \tag{A.7}$$

where the first surface term is the usual Gibbons-Hawking term, and the second is a natural generalization of (3.8) to arbitrary dimension. We will see that for AdS invariant boundary conditions this gives the surface term required for the hamiltonian to have well defined functional derivatives. To simplify the calculation of the volume term in the action we can use a scaling argument similar to the one in section 2. If we rescale the metric by a constant, the volume term becomes

$$I(\lambda^2 g) = \lambda^{d-1}\int\left[-\frac{1}{2}R + \frac{1}{2}(\nabla\phi)^2\right] + \lambda^{d+1}\int V(\phi)\,. \tag{A.8}$$

The instanton must be an extremum of the action, and since the surface terms don't contribute to the equations of motion, it must extremize the volume term. Setting $dI/d\lambda|_{\lambda=1} = 0$ we learn that

$$\int\left[-\frac{1}{2}R + \frac{1}{2}(\nabla\phi)^2\right] = -\frac{d+1}{d-1}\int V(\phi)\,. \tag{A.9}$$

Hence

$$I = -\frac{2}{d-1}\int V(\phi) - \oint K + \frac{1}{2d}\oint \left[(\nabla\phi)^2 - m^2\phi^2\right]. \tag{A.10}$$

The divergences can simply be determined by evaluating (A.10) using the asymptotic behavior (A.4)-(A.6) of the instantons. This gives

$$\begin{aligned}
-\frac{2}{d-1}\int V(\phi) &= \mathrm{Vol}(S^{d-1})\int^{\rho_0} d\rho\rho^{d-1}\left[d - \frac{d}{2\rho^2} - \frac{(2m^2+d\Delta_-)\alpha^2}{2(d-1)\rho^{2\Delta_-}}\right] \\
-\oint K &= -d\mathrm{Vol}(S^{d-1})\rho_0^d\left(1+\frac{1}{2\rho_0^2}+\frac{\Delta_-\alpha^2}{2(d-1)\rho_0^{2\Delta_-}}\right) \\
\frac{1}{2d}\oint\left[(\nabla\phi)^2 - m^2\phi^2\right] &= \mathrm{Vol}(S^{d-1})\frac{\Delta_-\alpha^2}{2}\rho_0^{d-2\Delta_-}.
\end{aligned} \tag{A.11}$$

The relevant quantity for computing the rate of vacuum decay is the difference between the instanton action and the action for pure AdS: $\Delta I = I - I_{\mathrm{AdS}}$. If ΔI were infinite the probability for the vacuum to decay would be zero. The background action I_{AdS} diverges as

$$I_{\mathrm{AdS}} = d\mathrm{Vol}(S^{d-1})\left[\int^{\rho_0} d\rho\rho^{d-1}\left(1-\frac{1}{2\rho^2}\right) - \rho_0^d\left(1+\frac{1}{2\rho_0^2}\right)\right]. \tag{A.12}$$

Hence subtracting I_{AdS} removes the leading divergences in I, but since ϕ goes to zero so slowly there is a subleading divergence. However, by using (A.5) one sees the coefficient of the divergent terms add up to zero. This involves nontrivial cancellations among the volume term and both surface terms in the action. A second, logarithmic divergence coming from $\int V$ does not appear due to cancellations between the potential and volume element. The difference $\Delta I = I - I_{\mathrm{AdS}}$, therefore, is finite, yielding a non-zero decay rate. Note that ΔI is finite for all α and β; we have not yet assumed any relation between them.

If one analytically continues the instanton, one obtains solutions that evolve to a big crunch. Since one often uses the boundary conditions (A.4) to compute correlation functions of operators in the CFT, one might wonder if all of these theories are unstable. The answer is no. If one starts with the standard quantization, $\alpha = 0$, then adding a nonzero α corresponds to adding a term proportional to α to the CFT action. In this context one usually assumes α is a constant, but we have seen in section 3.5 that the analytic continuation of the instanton corresponds to an α which is time dependent and diverges in finite time. So using the standard boundary conditions, the big crunch would correspond to adding an explicitly diverging term to the action. In addition, once one modifies the CFT action, the usual vacuum AdS_{d+1} is no longer a solution since the field is required to satisfy (A.4) with nonzero α.

By contrast, AdS invariant boundary conditions exist in all dimensions [6]. Asymptotic AdS invariance only requires $\beta = f\alpha^{\Delta_+/\Delta_-}$, where f is an arbitrary constant. So α and β are related, but can be zero or time dependent. In general there is a contribution from the scalar field to the conserved charges,

$$Q[\xi] = Q_G[\xi] + \frac{1}{2d}\oint \xi^\perp\left[(\nabla\phi)^2 - m^2\phi^2\right], \tag{A.13}$$

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where $Q_G[\xi]$ is the standard gravitational surface term (3.5). For spherical solutions this gives

$$Q[\partial_t] = \mathrm{Vol}(S^{d-1})\left(\frac{d-1}{2}M_0 - \frac{2fm^2\alpha^{d/\Delta_-}}{d}\right), \tag{A.14}$$

where M_0 is the coefficient of the $O(1/r^{d+2})$ correction to the g_{rr} component of the AdS metric. Hence the gravitational contribution $Q_G[\partial_t]$ to the mass of the instantons can be read off from (A.6). One sees it diverges as $\rho^{d-2\Delta_-}$. However this divergence, as well as the finite gravitational contribution proportional to M_0, are exactly cancelled by the scalar contributions. Instantons of this type thus specify nontrivial, zero mass initial data for all m^2 in the range $m^2_{BF} < m^2 < m^2_{BF}+1$ and in all dimensions $d+1$. In general, the CFT duals of these AdS invariant boundary conditions are not yet well understood. We discussed the simplest case in section 3.

Finally we turn to the case of a scalar field with $m^2 = m^2_{BF}$. For fields that saturate the BF bound, $\Delta_+ = \Delta_- = d/2$ and the second solution asymptotically behaves like $\ln\rho/\rho^{d/2}$. Thus asymptotically we have

$$\phi = \frac{\alpha\ln\rho}{\rho^{d/2}} + \frac{\beta}{\rho^{d/2}} \tag{A.15}$$

and therefore

$$b^2 = \rho^2 + 1 + \frac{d\alpha^2(\ln\rho)^2}{2(d-1)\rho^{d-2}} + \frac{\alpha(d\beta-\alpha)\ln\rho}{(d-1)\rho^{d-2}} + \frac{d^2\beta^2 - 2d\alpha\beta + 2\alpha^2}{2d(d-1)\rho^{d-2}}\,. \tag{A.16}$$

Asymptotic solutions of this form can similarly be regarded as having AdS invariant boundary conditions provided $\beta = \alpha(f - \frac{2}{d}\ln\alpha)$ [6, 8]. As before, the scalar surface term (A.13) in the hamiltonian gives in general a non-vanishing contribution to the conserved charges. For instance, the total mass of spherically symmetric solutions is given by

$$\begin{aligned} Q[\partial_t] = \mathrm{Vol}\left(S^{d-1}\right)\Bigg[&\frac{d-1}{2}M_0 + \frac{1}{d}\alpha^2(\ln\alpha)^2 + \alpha^2\left(\frac{1}{d}-f\right)\ln\alpha + \\ &+ \frac{\alpha^2}{2}\left(\frac{d}{2}f^2 - f + \frac{1}{d}\right)\Bigg]. \end{aligned} \tag{A.17}$$

It is easily seen from (A.16) that the instantons have zero mass. Evaluating the action (A.10) gives

$$\begin{aligned} -\frac{2}{d-1}\int V(\phi) &= \mathrm{Vol}(S^{d-1})\int^{\rho_0}\frac{d\rho}{\rho}\left[d\rho^d - \frac{d}{2}\rho^{d-2} + \frac{d\alpha^2\ln\rho + d\alpha\beta - \alpha^2}{2(d-1)}\right] \\ -\oint K &= -\mathrm{Vol}(S^{d-1})\left(d\rho_0^d + \frac{d}{2}\rho_0^{d-2} + \frac{d^2\alpha^2\ln^2\rho_0 + 2d\alpha(d\beta-\alpha)\ln\rho_0}{4(d-1)}\right) \\ \oint\left[\frac{d}{8}\phi^2 + \frac{(\nabla\phi)^2}{2d}\right] &= \frac{\mathrm{Vol}(S^{d-1})}{4}\left[d\alpha^2\ln^2\rho_0 + 2(d\alpha\beta - \alpha^2)\ln\rho_0\right]. \end{aligned} \tag{A.18}$$

Subtracting the background action (A.12) and using (A.5) again yields a finite result. The value of the euclidean action determines the rate of vacuum decay. The analytic continuation of the instantons again provides examples of smooth initial data which evolve to a big crunch.

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References

[1] J.B. Hartle and S.W. Hawking, *Wave function of the universe*, *Phys. Rev.* **D 28** (1983) 2960.

[2] M. Gasperini and G. Veneziano, *Pre-big bang in string cosmology*, *Astropart. Phys.* **1** (1993) 317 [hep-th/9211021]; *The pre-big bang scenario in string cosmology*, *Phys. Rept.* **373** (2003) 1 [hep-th/0207130].

[3] J. Khoury, B.A. Ovrut, N. Seiberg, P.J. Steinhardt and N. Turok, *From big crunch to big bang*, *Phys. Rev.* **D 65** (2002) 086007 [hep-th/0108187];
P.J. Steinhardt and N. Turok, *Cosmic evolution in a cyclic universe*, *Phys. Rev.* **D 65** (2002) 126003 [hep-th/0111098].

[4] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[5] L. Fidkowski, V. Hubeny, M. Kleban and S. Shenker, *The black hole singularity in AdS/CFT*, *J. High Energy Phys.* **02** (2004) 014 [hep-th/0306170].

[6] T. Hertog and K. Maeda, *Black holes with scalar hair and asymptotics in $N = 8$ supergravity*, *J. High Energy Phys.* **07** (2004) 051 [hep-th/0404261].

[7] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Ann. Phys. (NY)* **144** (1982) 249; *Positive energy in anti-de Sitter backgrounds and gauged extended supergravity*, *Phys. Lett.* **B 115** (1982) 197.

[8] M. Henneaux, C. Martinez, R. Troncoso and J. Zanelli, *Asymptotically anti-de Sitter spacetimes and scalar fields with a logarithmic branch*, hep-th/0404236.

[9] T. Hertog, G.T. Horowitz and K. Maeda, *Negative energy in string theory and cosmic censorship violation*, *Phys. Rev.* **D 69** (2004) 105001 [hep-th/0310054].

[10] M. Gutperle and P. Kraus, *Numerical study of cosmic censorship in string theory*, *J. High Energy Phys.* **04** (2004) 024 [hep-th/0402109].

[11] V.E. Hubeny, X. Liu, M. Rangamani and S. Shenker, *Comments on cosmic censorship in AdS/CFT*, hep-th/0403198.

[12] J.M. Maldacena, J. Michelson and A. Strominger, *Anti-de Sitter fragmentation*, *J. High Energy Phys.* **02** (1999) 011 [hep-th/9812073].

[13] M. Heusler, N. Straumann, *Scaling arguments for the existence of static, spherically symmetric solutions of self-gravitating systems*, *Class. and Quant. Grav.* **9** (1992) 2177;
M. Heusler, *A no hair theorem for selfgravitating nonlinear sigma models*, *J. Math. Phys.* **33** (1992) 3497.

[14] D. Sudarsky and R.M. Wald, *Extrema of mass, stationarity and staticity and solutions to the Einstein Yang-Mills equations*, *Phys. Rev.* **D 46** (1992) 1453.

[15] B. de Wit and H. Nicolai, *$N = 8$ supergravity with local $\mathrm{SO}(8) \times \mathrm{SU}(8)$ invariance*, *Phys. Lett.* **B 108** (1982) 285; *N=8 supergravity*, *Nucl. Phys.* **B 208** (1982) 323.

[16] M.J. Duff and J.T. Liu, *Anti-de Sitter black holes in gauged $N = 8$ supergravity*, *Nucl. Phys.* **B 554** (1999) 237 [hep-th/9901149].

[17] L.F. Abbott and S. Deser, *Stability of gravity with a cosmological constant*, *Nucl. Phys.* **B 195** (1982) 76.

JHEP07(2004)073

[18] M. Henneaux and C. Teitelboim, *Asymptotically anti-de Sitter spaces*, *Commun. Math. Phys.* **98** (1985) 391.

[19] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[20] S. de Haro, S.N. Solodukhin and K. Skenderis, *Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence*, *Commun. Math. Phys.* **217** (2001) 595 [hep-th/0002230].

[21] M. Bianchi, D.Z. Freedman and K. Skenderis, *Holographic renormalization*, *Nucl. Phys.* **B 631** (2002) 159 [hep-th/0112119].

[22] M. Dafermos, *A note on naked singularities and the collapse of self-gravitating Higgs fields*, gr-qc/0403033.

[23] S.R. Coleman and F. De Luccia, *Gravitational effects on and of vacuum decay*, *Phys. Rev.* **D 21** (1980) 3305.

[24] O. Aharony, Y. Oz and Z. Yin, *M-theory on $AdS_p \times S^{11-p}$ and superconformal field theories*, *Phys. Lett.* **B 430** (1998) 87 [hep-th/9803051].

[25] E. Witten, *Multi-trace operators, boundary conditions and AdS/CFT correspondence*, hep-th/0112258.

[26] S.S. Gubser and I.R. Klebanov, *A universal result on central charges in the presence of double-trace deformations*, *Nucl. Phys.* **B 656** (2003) 23 [hep-th/0212138].

[27] S.S. Gubser and I. Mitra, *Double-trace operators and one-loop vacuum energy in AdS/CFT*, *Phys. Rev.* **D 67** (2003) 064018 [hep-th/0210093].

[28] M. Günaydin, L.J. Romans and N.P. Warner, *Gauged $N = 8$ supergravity in five-dimensions*, *Phys. Lett.* **B 154** (1985) 268; *Compact and noncompact gauged supergravity theories in five dimensions*, *Nucl. Phys.* **B 272** (1986) 598.

[29] M. Pernici, K. Pilch and P. van Nieuwenhuizen, *Gauged $N = 8$ $D = 5$ supergravity*, *Nucl. Phys.* **B 259** (1985) 460.

[30] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of D3-branes and gauged supergravity*, *J. High Energy Phys.* **07** (2000) 038 [hep-th/9906194].

[31] G.W. Gibbons, C.M. Hull and N.P. Warner, *The stability of gauged supergravity*, *Nucl. Phys.* **B 218** (1983) 173.

[32] T. Hertog, G.T. Horowitz and K. Maeda, *Generic cosmic censorship violation in anti de Sitter space*, *Phys. Rev. Lett.* **92** (2004) 131101 [gr-qc/0307102].

[33] T. Hertog, G.T. Horowitz and K. Maeda, *Update on cosmic censorship violation in AdS*, gr-qc/0405050.

[34] P.K. Townsend, *Positive energy and the scalar potential in higher dimensional (super)gravity theories*, *Phys. Lett.* **B 148** (1984) 55.